

On Capacity Scaling of Underwater Networks: An Information-Theoretic Perspective

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Abstract

Capacity scaling laws are analyzed in an underwater acoustic network with n regularly located nodes on a square. A narrow-band model is assumed where the carrier frequency is allowed to scale as a function of n . In the network, we characterize an attenuation parameter that depends on the frequency scaling as well as the transmission distance. A cut-set upper bound on the throughput scaling is then derived in extended networks. Our result indicates that the upper bound is inversely proportional to the attenuation parameter, thus resulting in a highly power-limited network. Interestingly, it is seen that unlike the case of wireless radio networks, our upper bound is intrinsically related to the attenuation parameter but not the spreading factor. Furthermore, we describe an achievable scheme based on the simple nearest-neighbor multi-hop (MH) transmission. It is shown under extended networks that the MH scheme is order-optimal as the attenuation parameter scales exponentially with \sqrt{n} (or faster). Finally, these scaling results are extended to a random network realization.

Index Terms

Attenuation parameter, capacity scaling law, carrier frequency, cut-set upper bound, extended network, multi-hop (MH), power-limited, underwater acoustic network.

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I. INTRODUCTION

A pioneering work of [1], introduced by Gupta and Kumar, characterized the sum throughput scaling in a large wireless radio network. They showed that the total throughput scales as $\Theta(\sqrt{n/\log n})$ when a multi-hop (MH) routing strategy is used for n source–destination (S–D) pairs randomly distributed in a unit area.¹ MH schemes are then further developed and analyzed in [3]–[9], while their throughput per S–D pair scales far slower than $\Theta(1)$. Recent results [10], [11] have shown that an almost linear throughput in the radio network, i.e. $\Theta(n^{1-\epsilon})$ for an arbitrarily small $\epsilon > 0$, which is the best we can hope for, is achievable by using a hierarchical cooperation strategy. Besides the schemes in [10], [11], there have been other studies to improve the throughput of wireless radio networks up to a linear scaling in a variety of network scenarios by using novel techniques such as networks with node mobility [12], interference alignment [13], and infrastructure support [14].

Along with the studies in terrestrial radio networks, the interest in study of underwater networks has been growing with recent advances in acoustic communication technology [15]–[18]. In underwater acoustic communication systems, both bandwidth and power are severely limited due to the exponential (rather than polynomial) path-loss attenuation with propagation distance and even frequency-dependent attenuation. This is a main feature that distinguishes underwater systems from wireless radio links. Hence, the system throughput is affected by not only the transmission distance but also the useful bandwidth. Based on these characteristics, network coding schemes [17], [19], [20] have been presented for underwater acoustic channels, while network coding showed better performance than MH routing in terms of reducing transmission power. MH networking has further been investigated in other simple but realistic network conditions that take into account the practical issues of coding and delay [21], [22].

One natural question is what are the fundamental capabilities of underwater networks in supporting a multiplicity of nodes that wish to communicate concurrently with each other, i.e., multiple S–D pairs, over an acoustic channel. To answer this question, the throughput scaling for underwater networks was first studied [23], where n nodes were arbitrarily located in a planar disk of unit area [1] and the carrier frequency was set to a constant independent of n . That work showed an upper bound on the throughput of each node based on the physical model assumption in [1]. This upper bound scales as $n^{-1/\alpha} e^{-W_0(\Theta(n^{-1/\alpha}))}$, where α corresponds to the spreading factor of the underwater channel, and W_0 represents the branch zero of the Lambert function [24].² Since the spreading factor typically has values in the range $1 \leq \alpha \leq 2$ [23], the throughput per node decreases almost as $O(n^{-1/\alpha})$ for large enough n , which is considerably faster than the $\Theta(\sqrt{n})$ scaling characterized for wireless radio settings [1].

In this paper, a capacity scaling law for underwater networks is analyzed in extended networks [4], [5], [10], [25], [26] of unit node density. Especially, we are interested in the case where the carrier frequency scales as a certain function of n in a narrow-band model. Such an assumption changes the scaling behavior significantly due to the attenuation characteristics. We aim to study both an information-theoretic upper bound and achievable rate scaling while allowing the frequency scaling with n .

We explicitly characterize an *attenuation parameter* that depends on the transmission distance and also on the carrier frequency, and then identify fundamental operating regimes depending on the parameter. For networks with n regularly distributed nodes, we derive an upper bound on the total throughput scaling using the cut-set bound. Our upper bound is based on the characteristics of power-limited regimes shown in [10], [27]. In extended networks, it is shown that the upper bound is inversely proportional to the attenuation parameter. This leads to a highly power-limited network for all the operating regimes, where power consumption is important in determining performance. Interestingly, it is seen that unlike the case of wireless radio networks, our upper bound heavily depends on the attenuation parameter but

¹We use the following notations: i) $f(x) = O(g(x))$ means that there exist constants C and c such that $f(x) \leq Cg(x)$ for all $x > c$. ii) $f(x) = o(g(x))$ means that $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$. iii) $f(x) = \Omega(g(x))$ if $g(x) = O(f(x))$. iv) $f(x) = \omega(g(x))$ if $g(x) = o(f(x))$. v) $f(x) = \Theta(g(x))$ if $f(x) = O(g(x))$ and $g(x) = O(f(x))$ [2].

²The Lambert W function is defined to be the inverse of the function $z = W(z)e^{W(z)}$ and the branch satisfying $W(z) \geq -1$ is denoted by $W_0(z)$.

not on the spreading factor (corresponding to the path-loss exponent in wireless networks). In addition, to constructively show our achievability result for extended regular networks, we describe the conventional nearest-neighbor MH transmission [1] with a slight modification, and analyze its achievable throughput. It is shown under extended networks that the achievable rate based on the MH routing scheme matches the upper bound within a factor of n with arbitrarily small exponent as long as the attenuation parameter increases exponentially with respect to \sqrt{n} (or faster). Furthermore, a random network scenario is also discussed in this work. It is shown under extended random networks that the conventional MH-based achievable scheme is not order-optimal for any operating regimes.

The rest of this paper is organized as follows. Section II describes our system and channel models. In Section III, the cut-set upper bound on the throughput is derived. In Section IV, achievable throughput scaling is analyzed. These results are extended to the random network case in Section V. Finally, Section VI summarizes the paper with some concluding remarks.

Throughout this paper the superscript H , $[\cdot]_{ki}$, and $\|\cdot\|_2$ denote the conjugate transpose, the (k, i) -th element, and the largest singular value, respectively, of a matrix. \mathbf{I}_n is the identity matrix of size $n \times n$, $\text{tr}(\cdot)$ is the trace, and $\det(\cdot)$ is the determinant. \mathbb{C} is the field of complex numbers and $E[\cdot]$ is the expectation. Unless otherwise stated, all logarithms are assumed to be to the base 2.

II. SYSTEM AND CHANNEL MODELS

We consider a two-dimensional underwater network that consists of n nodes on a square with unit node density such that two neighboring nodes are 1 unit of distance apart from each other in an extended network³, i.e., a regular network [25], [26]. We randomly pick a matching of S–D pairs, so that each node is the destination of exactly one source. We assume frequency-flat channel of bandwidth W Hz around carrier frequency f , which satisfies $f \gg W$, i.e., narrow-band model. This is a highly simplified model, but nonetheless one that suffices to demonstrate the fundamental mechanisms that govern capacity scaling. Assuming that all the nodes have perfectly directional transmissions, we also disregard multipath propagation, and simply focus on a line-of-sight channel between each pair of nodes used in [10], [11], [27]. Each node has an average transmit power constraint P (constant), and we assume that the channel state information (CSI) is available at all receivers, but not at the transmitters. It is assumed that each node transmits at a rate $T(n)/n$, where $T(n)$ denotes the total throughput of the network.

Now let us turn to channel modeling. An underwater acoustic channel is characterized by an attenuation that depends on both the distance r_{ki} between nodes i and k ($i, k \in \{1, \dots, n\}$) and the signal frequency f , and is given by

$$A(r_{ki}, f) = c_0 r_{ki}^\alpha a(f)^{r_{ki}} \quad (1)$$

for some constant $c_0 > 0$ independent of n , where α is the spreading factor and $a(f) > 1$ is the absorption coefficient [16]. For analytical convenience, we assume that the spreading factor α does not change throughout the network, i.e., that it is the same from short to long range transmissions, as in wireless radio networks [1], [4], [10]. The spreading factor describes the geometry of propagation and is typically $1 \leq \alpha \leq 2$ —its commonly used values are $\alpha = 1$ for cylindrical spreading, $\alpha = 2$ for spherical spreading, and $\alpha = 1.5$ for the so-called practical spreading. Note that existing models of wireless networks typically correspond to the case for which $a(f) = 1$ (or a positive constant independent of n) and $\alpha > 2$.⁴

A common empirical model gives $a(f)$ in dB/km for f in kHz as [16], [28]:

$$10 \log a(f) = a_0 + a_1 f^2 + a_2 \frac{f^2}{b_1 + f^2} + a_3 \frac{f^2}{b_2 + f^2}, \quad (2)$$

³A dense network [1], [6], [10] of unit area can also be considered as another fundamental network model, which will not be shown in this work. We remark that there exists either a bandwidth or power limitation (or both) according to the path-loss attenuation regimes in dense networks.

⁴The counterpart of α in wireless radio channels is the path-loss exponent.

where $\{a_0, \dots, a_3, b_1, b_2\}$ are some positive constants independent of n . As mentioned earlier, we will allow the carrier frequency f to scale with n . Especially, we consider the case where the frequency scales at arbitrarily increasing rates relative to n , which enables us to really capture the dependence on the frequency in performance.⁵ The absorption $a(f)$ is then an increasing function of f such that

$$a(f) = \Theta\left(e^{c_1 f^2}\right) \quad (3)$$

with respect to f for some constant $c_1 > 0$ independent of n .

The noise n_i at node $i \in \{1, \dots, n\}$ in an acoustic channel can be modeled through four basic sources: turbulence, shipping, waves, and thermal noise [16]. We assume that n_i is the circularly symmetric complex additive colored Gaussian noise with zero mean and power spectral density (psd) $N(f)$, and thus the noise is frequency-dependent. The overall psd of four sources decays linearly on the logarithmic scale in the frequency region 100 Hz – 100 kHz, which is the operating regime used by the majority of acoustic systems, and thus is approximately given by [16], [29]

$$\log N(f) = a_4 - a_5 \log f \quad (4)$$

for some positive constants a_4 and a_5 independent of n .⁶ This means that $N(f) = O(1)$ since

$$N(f) = \Theta\left(\frac{1}{f^{a_5}}\right) \quad (5)$$

in terms of f increasing with n . From (3) and (5), we may then have the following relationship between the absorption $a(f)$ and the noise psd $N(f)$:

$$N(f) = \Theta\left(\frac{1}{(\log a(f))^{a_5/2}}\right). \quad (6)$$

From the narrow-band assumption, the received signal y_k at node $k \in \{1, \dots, n\}$ at a given time instance is given by

$$y_k = \sum_{i \in I} h_{ki} x_i + n_k, \quad (7)$$

where

$$h_{ki} = \frac{e^{j\theta_{ki}}}{\sqrt{A(r_{ki}, f)}} \quad (8)$$

represents the complex channel between nodes i and k , $x_i \in \mathbb{C}$ is the signal transmitted by node i , and $I \subset \{1, \dots, n\}$ is the set of simultaneously transmitting nodes. The random phases $e^{j\theta_{ki}}$ are uniformly distributed over $[0, 2\pi)$ and independent for different i , k , and time. We thus assume a narrow-band time-varying channel, whose gain changes to a new independent value for every symbol. Note that this random phase model is based on a far-field assumption [10], [11], [27],⁷ which is valid if the wavelength is sufficiently smaller than the minimum node separation.

Based on the above channel characteristics, operating regimes of the network are identified according to the following physical parameters: the absorption $a(f)$ and the noise psd $N(f)$ which are exploited here by choosing the frequency f based on the number n of nodes. In other words, if the relationship between f and n is specified, then $a(f)$ and $N(f)$ can be given by a certain scaling function of n from (3) and (5), respectively.

⁵Otherwise, the attenuation parameter $a(f)$ scales as $\Theta(1)$ from (2), which is not a matter of interest in this work.

⁶Note that in our operating frequencies, $a_5 = 1.8$ is commonly used for the above approximation [16].

⁷In [30], instead of simply taking the far-field assumption, the physical limit of wireless radio networks has been studied under certain conditions on scattering elements. Further investigation is also required to see whether this assumption is valid for underwater networks of unit node density in the limit of large number n of nodes.

III. CUT-SET UPPER BOUND

To access the fundamental limits of an underwater network, a cut-set upper bound on the total throughput scaling is analyzed from an information-theoretic perspective [31]. Specifically, an upper bound based on the power transfer argument [10], [27] is established for extended networks. Note, however, that the present problem is not equivalent to the conventional extended network framework [10] due to different channel characteristics. Our interest is particularly in the operating regimes for which the upper bound is tight.

Consider a given cut L dividing the network area into two equal halves as in [10], [27] (see Fig. 1). Let S_L and D_L denote the sets of sources and destinations, respectively, for the cut L in the network. More precisely, under L , source nodes S_L are on the left, while all nodes on the right are destinations D_L . In this case, we have an $\Theta(n) \times \Theta(n)$ multiple-input multiple-output (MIMO) channel between the two sets of nodes separated by the cut.

In an extended network, we take into account an approach based on the amount of power transferred across the network according to different operating regimes, i.e., path-loss attenuation regimes. As pointed out in [10], [27], the information transfer from S_L to D_L is highly power-limited since all the nodes in the set D_L are ill-connected to the left-half network in terms of power. This implies that the information transfer is bounded by the total received power transfer, rather than the cardinality of the set D_L . For the cut L , the total throughput $T(n)$ for sources on the left is bounded by the (ergodic) capacity of the MIMO channel between S_L and D_L under time-varying channel assumption, and thus is given by

$$T(n) \leq \max_{\mathbf{Q}_L \geq 0} E \left[\log \det \left(\mathbf{I}_{\Theta(n)} + \frac{1}{N(f)} \mathbf{H}_L \mathbf{Q}_L \mathbf{H}_L^H \right) \right], \quad (9)$$

where \mathbf{H}_L is the matrix with entries $[\mathbf{H}_L]_{ki}$ for $i \in S_L, k \in D_L$, and $\mathbf{Q}_L \in \mathbb{C}^{\Theta(n) \times \Theta(n)}$ is the positive semi-definite input signal covariance matrix whose k -th diagonal element satisfies $[\mathbf{Q}_L]_{kk} \leq P$ for $k \in S_L$.

The relationship (9) will be further specified in Theorem 1. Before that, we first apply the techniques of [10], [32] to obtain the total power transfer of the set D_L . These techniques involve the relaxation of the individual power constraints to a total weighted power constraint, where the weight assigned to each source corresponds to the total received power on the other side of the cut. To be more specific, each column i of the matrix \mathbf{H}_L is normalized by the square root of the total received power on the other side of the cut from source $i \in S_L$. From (1) and (8), the total power $P_L^{(i)}$ received from the signal sent by the source i is given by

$$P_L^{(i)} = P d_L^{(i)}, \quad (10)$$

where

$$d_L^{(i)} = \frac{1}{c_0} \sum_{k \in D_L} r_{ki}^{-\alpha} a(f)^{-r_{ki}} \quad (11)$$

for some constant $c_0 > 0$ independent of n . For convenience, we now index the node positions such that the source and destination nodes under the cut L are located at positions $(-i_x + 1, i_y)$ and (k_x, k_y) , respectively, for $i_x, k_x = 1, \dots, \sqrt{n}/2$ and $i_y, k_y = 1, \dots, \sqrt{n}$. The scaling result of $d_L^{(i)}$ defined in (11) can then be derived as follows.

Lemma 1: In an extended network, the term $d_L^{(i)}$ in (11) is

$$d_L^{(i)} = \Theta \left(i_x^{1-\alpha} a(f)^{-i_x} \right), \quad (12)$$

where $-i_x + 1$ represents the horizontal coordinate of node $i \in S_L$ for $i_x = 1, \dots, \sqrt{n}/2$.

The proof of this lemma is presented in Appendix A. The expression (9) is then rewritten as

$$\max_{\tilde{\mathbf{Q}}_L \geq 0} E \left[\log \det \left(\mathbf{I}_{\Theta(n)} + \frac{1}{N(f)} \mathbf{F}_L \tilde{\mathbf{Q}}_L \mathbf{F}_L^H \right) \right], \quad (13)$$

where \mathbf{F}_L is the matrix with entries $[\mathbf{F}_L]_{ki} = \frac{1}{\sqrt{d_L^{(i)}}} [\mathbf{H}_L]_{ki}$, which are obtained from (11), for $i \in S_L, k \in D_L$. Here, $\tilde{\mathbf{Q}}_L$ is the matrix satisfying

$$[\tilde{\mathbf{Q}}_L]_{ki} = \sqrt{d_L^{(k)} d_L^{(i)}} [\mathbf{Q}_L]_{ki},$$

which means that $\text{tr}(\tilde{\mathbf{Q}}_L) \leq \sum_{i \in S_L} P_L^{(i)}$ (equal to the sum of the total power received from each source).

We next examine the behavior of the largest singular value for the normalized channel matrix \mathbf{F}_L , and then show how much it affects an upper bound on (13). We first address the case where \mathbf{F}_L is well-conditioned according to the attenuation parameter $a(f)$.

Lemma 2: Let \mathbf{F}_L denote the normalized channel matrix defined by the expression (13). Under the attenuation regimes $a(f) = \Omega((1 + \epsilon_0)^{\sqrt{n}})$ for an arbitrarily small $\epsilon_0 > 0$, we have that

$$E [\|\mathbf{F}_L\|_2^2] \leq c_2 \log n \quad (14)$$

for some constant $c_2 > 0$ independent of n .

The proof of this lemma is presented in Appendix B. Note that the matrix \mathbf{F}_L is well-conditioned as $a(f)$ scales exponentially with respect to \sqrt{n} (or faster). Otherwise, i.e., if $a(f) = o((1 + \epsilon_0)^{\sqrt{n}})$, the largest singular value of \mathbf{F}_L scales as a polynomial factor of n , thus resulting in a loose upper bound on the total throughput. Using Lemma 2, we obtain the following result.

Lemma 3: Under $a(f) = \Omega((1 + \epsilon_0)^{\sqrt{n}})$, the term (13) is upper-bounded by

$$\frac{n^\epsilon}{N(f)} \sum_{i \in S_L} P_L^{(i)} \quad (15)$$

for arbitrarily small positive constants ϵ_0 and ϵ , where $P_L^{(i)}$ is given by (10).

Proof: Equation (13) is bounded by

$$\begin{aligned} & \max_{\tilde{\mathbf{Q}}_L \geq 0} E \left[\log \det \left(\mathbf{I}_{\Theta(n)} + \frac{1}{N(f)} \mathbf{F}_L \tilde{\mathbf{Q}}_L \mathbf{F}_L^H \right) 1_{\mathcal{E}_{\mathbf{F}_L}} \right] \\ & + \max_{\tilde{\mathbf{Q}}_L \geq 0} E \left[\log \det \left(\mathbf{I}_{\Theta(n)} + \frac{1}{N(f)} \mathbf{F}_L \tilde{\mathbf{Q}}_L \mathbf{F}_L^H \right) 1_{\mathcal{E}_{\mathbf{F}_L}^c} \right]. \end{aligned} \quad (16)$$

Here, the event $\mathcal{E}_{\mathbf{F}_L}$ refers to the case where the channel matrix \mathbf{F}_L is accidentally ill-conditioned and is given by

$$\mathcal{E}_{\mathbf{F}_L} = \{\|\mathbf{F}_L\|_2^2 > n^\epsilon\}$$

for an arbitrarily small constant $\epsilon > 0$. Suppose that $a(f) = \Omega((1 + \epsilon_0)^{\sqrt{n}})$. Then, by using the result of Lemma 2 and applying the proof technique similar to that in Section V of [10], it is possible to prove that the first term in (16) decays polynomially to zero with arbitrary exponent as n tends to infinity, and for the second term in (16), it follows that

$$\begin{aligned} & \max_{\tilde{\mathbf{Q}}_L \geq 0} E \left[\log \det \left(\mathbf{I}_{\Theta(n)} + \frac{1}{N(f)} \mathbf{F}_L \tilde{\mathbf{Q}}_L \mathbf{F}_L^H \right) 1_{\mathcal{E}_{\mathbf{F}_L}^c} \right] \\ & \leq \max_{\tilde{\mathbf{Q}}_L \geq 0} E \left[\frac{1}{N(f)} \|\mathbf{F}_L\|_2^2 \text{tr}(\tilde{\mathbf{Q}}_L) 1_{\mathcal{E}_{\mathbf{F}_L}^c} \right] \\ & \leq \frac{c_2 \log n}{N(f)} \max_{\tilde{\mathbf{Q}}_L \geq 0} \text{tr}(\tilde{\mathbf{Q}}_L) \\ & \leq \frac{n^\epsilon}{N(f)} \sum_{i \in S_L} P_L^{(i)} \end{aligned}$$

for some constant $c_2 > 0$ independent of n , where the second inequality holds by Lemma 2. This completes the proof of this lemma. ■

Note that (15) represents the total amount of received signal-to-noise ratio from the set S_L of sources to the set D_L of destinations for a given cut L . We are now ready to show the cut-set upper bound in extended networks.

Theorem 1: For an underwater regular network of unit node density, where the absorption coefficient $a(f)$ scales as $\Omega((1 + \epsilon_0)^{\sqrt{n}})$ for an arbitrarily small $\epsilon_0 > 0$, the total throughput $T(n)$ is upper-bounded by

$$T(n) \leq \frac{c_3 n^{1/2+\epsilon}}{a(f)N(f)}, \quad (17)$$

where $c_3 > 0$ is some constant independent of n and $\epsilon > 0$ is an arbitrarily small constant.

Proof: Suppose that $a(f) = \Omega((1 + \epsilon_0)^{\sqrt{n}})$. Then from Lemmas 1 and 3, we obtain the following upper bound on the total throughput $T(n)$:

$$\begin{aligned} T(n) &\leq \frac{n^\epsilon}{N(f)} \sum_{i \in S_L} P d_L^{(i)} \\ &\leq \frac{P n^\epsilon}{N(f)} \sum_{i_x=1}^{\sqrt{n}/2} \sum_{i_y=1}^{\sqrt{n}} d_L^{(i)} \\ &\leq \frac{c_4 P n^{1/2+\epsilon}}{N(f)} \sum_{i_x=1}^{\sqrt{n}/2} \frac{1}{i_x^{\alpha-1} a(f)^{i_x}} \\ &\leq \frac{c_4 P n^{1/2+\epsilon}}{N(f)} \sum_{i_x=1}^{\sqrt{n}/2} \frac{1}{a(f)^{i_x}} \\ &\leq \frac{c_5 P n^{1/2+\epsilon}}{a(f)N(f)}, \end{aligned}$$

where c_4 and c_5 are some positive constants independent of n , which is equal to (17). This completes the proof of the theorem. ■

Note that this upper bound is expressed as a function of the absorption $a(f)$ and the noise psd $N(f)$ while an upper bound for wireless radio networks depends only on the constant value α [10]. We remark that when $a(f) = o((1 + \epsilon_0)^{\sqrt{n}})$, the upper bound becomes boosted by a certain polynomial factor of n (up to $O(\sqrt{n})$) compared to the case shown in (17).⁸ In addition, using (6) in (17) results in

$$T(n) = O\left(\frac{(\log a(f))^{a_5/2} n^{1/2+\epsilon}}{a(f)}\right)$$

for some constant $a_5 > 0$ shown in (4). Finally, another expression for the condition in which the upper bound in (17) holds is shown as follows.

Remark 1: We examine the relationship between the carrier frequency f and the number n of nodes such that our upper bound holds. By using (3) and the regimes $a(f) = \Omega((1 + \epsilon_0)^{\sqrt{n}})$, we can also obtain the following condition:

$$f = \Omega(n^{1/4}),$$

which means that if f scales faster than $n^{1/4}$, then the result in (17) is satisfied.

⁸This statement could be rigorously proved by following the steps similar to those shown in Lemmas 2 and 3, even if the details are not shown in this paper.

IV. ACHIEVABILITY RESULT

In this section, we show that the considered transmission scheme, commonly used in wireless radio networks, is order-optimal in underwater networks. Under a regular network of unit node density, the conventional MH transmission is used and its achievable throughput scaling is analyzed to show its order optimality.

The nearest-neighbor MH routing protocol [1] will be briefly described with a slight modification. The basic procedure of the MH protocol under our extended regular network is as follows:

- Divide the network into square routing cells, each of which has unit area.
- Draw a line connecting a S–D pair. A source transmits a packet to its destination using the nodes in the adjacent cells passing through the line.
- The full power is used, i.e., the transmit powers at each node is P .

Instead of original (continuous) MH transmissions, a bursty transmission scheme [10], [27], which uses only a fraction $1/a(f)N(f)$ of the time for actual transmission with instantaneous power $a(f)N(f)P$ per node, is used to simply apply the analysis for networks with no power limitation to our network model. With this scheme, the received signal power from the desired transmitter, the noise psd, and the total interference power from the set $I \subset \{1, \dots, n\}$ have the same scaling, i.e., $\Theta(N(f))$, and the (instantaneous) received signal-to-interference-and-noise ratio (SINR) is kept at $\Theta(1)$ under the narrow-band model, which will be obviously shown in the proof of Theorem 2.

The achievable rate of MH is now shown by quantifying the amount of interference.

Lemma 4: Suppose that a regular network of unit node density uses the MH protocol with burstiness. Then, the total interference power from other simultaneously transmitting nodes, corresponding to the set $I \subset \{1, \dots, n\}$, is upper-bounded by $\Theta(N(f))$, where $N(f)$ denotes the psd of noise n_i at receiver $i \in \{1, \dots, n\}$.

Proof: There are $8k$ interfering routing cells, each of which includes one node, in the k -th layer l_k of the network as illustrated in Fig. 2. Then from (1), (7), and (8), the total interference power at each node from simultaneously transmitting nodes is upper-bounded by

$$\begin{aligned} \sum_{k=1}^{\infty} (8k) \frac{a(f)N(f)P}{c_0 k^{\alpha} a(f)^k} &= \frac{8N(f)P}{c_0} \sum_{k=1}^{\infty} \frac{1}{k^{\alpha-1} a(f)^{k-1}} \\ &\leq \frac{8N(f)P}{c_0} \sum_{k=1}^{\infty} \frac{1}{a(f)^{k-1}} \\ &\leq c_6 N(f), \end{aligned}$$

where c_0 and c_6 are some positive constants independent of n , which completes the proof. \blacksquare

Note that the signal power no longer decays polynomially but rather exponentially with propagation distance in our network. This implies that the absorption term $a(f)$ in (1) will play an important role in determining the performance. It is also seen that the upper bound on the total interference power does not depend on the spreading factor α . Using Lemma 4, it is now possible to simply obtain a lower bound on the capacity scaling in the network, and hence the following result presents the achievable rates under the MH protocol.

Theorem 2: In an underwater regular network of unit node density,

$$T(n) = \Omega\left(\frac{n^{1/2}}{a(f)N(f)}\right) \quad (18)$$

is achievable.

Proof: Suppose that only a fraction $1/a(f)N(f)$ of the time for actual transmission is used under the MH protocol with burstiness. Then, the SINR seen by any receiver is expressed as $\Omega(1)$ with an instantaneous transmit power $a(f)N(f)P$ since the total interference power is given by $O(N(f))$. Since

the Gaussian is the worst additive noise [33], [34], assuming it lower-bounds the throughput. Hence, by assuming full CSI at the receiver, the achievable throughput per S-D pair is lower-bounded by

$$\begin{aligned} & \frac{1}{a(f)N(f)} \log(1 + \text{SINR}) \\ & \geq \frac{1}{a(f)N(f)} \log \left(1 + \frac{N(f)P/c_0}{N(f) + c_6N(f)} \right), \end{aligned}$$

for some positive constants c_0 and c_6 independent of n , thereby providing the rate of

$$\Omega \left(\frac{1}{a(f)N(f)} \right).$$

Since the number of hops per S-D pair is given by $O(\sqrt{n})$, there exist $\Omega(\sqrt{n})$ source nodes that can be active simultaneously, and therefore the total throughput is finally given by (18), which completes the proof of the theorem. ■

Now it is examined how the upper bound shown in Section III is close to the achievable throughput scaling.

Remark 2: Based on Theorems 1 and 2, when $a(f) = \Omega((1 + \epsilon_0)\sqrt{n})$, i.e., $f = \Omega(n^{1/4})$, it is easy to see that the achievable rate and the upper bound are of the same order up to n^ϵ , where ϵ and ϵ_0 are vanishingly small positive constants. MH is therefore order-optimal in regular networks with unit node density under the above attenuation regimes.

We also remark that applying the hierarchical cooperation strategy [10] may not be helpful to improve the achievable throughput due to long-range MIMO transmissions, which severely degrade performance in highly power-limited networks.⁹ To be specific, at the top level of the hierarchy, the transmissions between two clusters having distance $O(\sqrt{n})$ become a bottleneck, and thus cause a significant throughput degradation. It is further seen that even with the random phase model, which may enable us to obtain enough degrees-of-freedom gain, the benefit of randomness cannot be exploited because of the power limitation.

V. EXTENSION TO RANDOM NETWORKS

In this section, we would like to mention a random network configuration, where n S-D pairs are uniformly and independently distributed on a square.

We first discuss an upper bound for extended networks of unit node density. A precise upper bound can be obtained using the binning argument of [10] (refer to Appendix V in [10] for the details). Consider the same cut L , which divides the network area into two halves, as that in the regular network case. For analytical convenience, we can artificially assume the empty zone E_L , in which there are no nodes in the network, consisting of a rectangular slab of width $0 < \bar{c} < \frac{1}{\sqrt{7}e^{1/4}}$, independent of n , immediately to the right of the centerline (cut), as done in [27] (see Fig. 3).¹⁰ Let us state the following lemma.

Lemma 5: Assume a two dimensional extended network where n nodes are uniformly distributed. When the network area is divided into n squares of unit area, there are fewer than $\log n$ nodes in each square with high probability.

Since the result in Lemma 5 depends on the node distribution but not the channel characteristics, the proof essentially follows that presented in [4]. By Lemma 5, we now take into account the network transformation resulting in a regular network with at most $\log n$ and $2 \log n$ nodes, on the left and right,

⁹In wireless radio networks of unit node density, the hierarchical cooperation provides a near-optimal throughput scaling for the operating regimes $2 < \alpha < 3$, where α denotes the path-loss exponent that is greater than 2 [10]. Note that the analysis in [10] is valid under the assumption that α is kept at the same value on all levels of hierarchy.

¹⁰ Although this assumption does not hold in our random configuration, it is shown in [27] that there exists a vertical cut such that there are no nodes located closer than $0 < \bar{c} < \frac{1}{\sqrt{7}e^{1/4}}$ on both sides of this cut when we allow a cut that is not necessarily linear. Such an existence is proved by using percolation theory [4], [35]. This result can be directly applied to our network model since it only relies on the node distribution but not the channel characteristics. Hence, removing the assumption does not cause any change in performance.

respectively, at each square vertex except for the empty zone (see Fig. 3). Then, the nodes in each square are moved together onto one vertex of the corresponding square. More specifically, under the cut L , the node displacement is performed in the sense of decreasing the Euclidean distance between source node $i \in S_L$ and the corresponding destination $k \in D_L$, as shown in Fig. 3, which will provide an upper bound on $d_L^{(i)}$ in (11). It is obviously seen that the amount of power transfer under the transformed regular network is greater than that under another regular network with at most $\log n$ nodes at each vertex, located at integer lattice positions in a square region of area n . Hence, the upper bound for random networks is boosted by at least a logarithmic factor of n compared to that of regular networks discussed in Section III.

Now we turn our attention to showing an achievable throughput for extended random networks. In this case, the nearest-neighbor MH protocol [1] can also be utilized since our network is highly power-limited. Then, the area of each routing cell needs to scale with $2 \log n$ to guarantee at least one node in a cell [1], [6].¹¹ Each routing cell operates based on 9-time division multiple access to avoid causing large interference to its neighboring cells [1], [6]. For the routing with continuous MH transmissions (i.e., no burstiness), since per-hop distance is given by $\Theta(\sqrt{\log n})$, the received signal power from the intended transmitter and the SINR seen by any receiver are expressed as

$$\frac{c_7 P}{(\log n)^{\alpha/2} a(f)^{c_8 \sqrt{\log n}}}$$

and

$$\Omega \left(\frac{1}{(\log n)^{\alpha/2} a(f)^{c_8 \sqrt{\log n}} N(f)} \right),$$

respectively, for some constants $c_7 > 0$ and $c_8 \geq \sqrt{2}$ independent of n . Since the number of hops per S-D pair is given by $O(\sqrt{n/\log n})$, there exist $\Omega(\sqrt{n/\log n})$ simultaneously active sources, and thus the total achievable throughput $T(n)$ is finally given by

$$T(n) = \Omega \left(\frac{n^{1/2}}{(\log n)^{(\alpha+1)/2} a(f)^{c_8 \sqrt{\log n}} N(f)} \right)$$

for some constant $c_8 \geq \sqrt{2}$ independent of n (note that this relies on the fact that $\log(1+x)$ can be approximated by x for small $x > 0$). Hence, using the MH protocol results in at least a polynomial decrease in the throughput compared to the regular network case shown in Section IV.¹² This comes from the fact that the received signal power tends to be mainly limited due to exponential attenuation with transmission distance $\Theta(\sqrt{\log n})$. Note that in underwater networks, randomness on the node distribution causes a huge performance degradation on the throughput scaling. Therefore, we may conclude that the existing MH scheme does not satisfy the order optimality under extended random networks regardless of the attenuation parameter $a(f)$.

VI. CONCLUSION

The attenuation parameter and the capacity scaling laws have been characterized in a narrow-band channel of underwater acoustic networks. Provided that the carrier frequency f scales at arbitrary rates relative to the number n of nodes, the information-theoretic upper bound and the achievable throughput were derived as a function of the attenuation parameter $a(f)$ in extended regular networks. Specifically, based on the power transfer argument, the upper bound was shown to decrease in inverse proportion to $a(f)$. In addition, to show the achievability result, the nearest-neighbor MH protocol was introduced

¹¹When methods from percolation theory are applied to our random network [4], [35], the routing area constructed during the highway phase is a certain positive constant that is less than 1 and independent of n . The distance in the draining and delivery phases, corresponding to the first and last hops of a packet transmission, is nevertheless given by some constant times $\log n$, thereby limiting performance, especially for the condition $a(f) = \omega(1)$. Hence, using the protocol in [4] indeed does not perform better than the conventional MH case [1] in random networks.

¹²In terrestrial radio channels, there is a logarithmic gap in the achievable scaling laws between regular and random networks [1], [25].

with a simple modification, and its throughput scaling was analyzed. We proved that the MH protocol is order-optimal as long as the frequency f scales faster than $n^{1/4}$. Our scaling results were also extended to the random network scenario, where it was shown that the conventional MH scheme does not satisfy the order optimality for all the operating regimes.

Suggestions for further research include, in dense networks of unit area, analyzing an upper bound and designing an achievable scheme whose throughput scaling is close to the upper bound.

APPENDIX

A. Proof of Lemma 1

Upper and lower bounds on $d_L^{(i)}$ can be found by using the node-indexing and layering techniques similar to those shown in Section VI of [32]. As illustrated in Fig. 4, layers are introduced, where the i -th layer l'_i of the network represents the ring with width 1 drawn based on a source node, whose coordinate is given by $(-i_x + 1, i_y)$, where $i \in \{1, \dots, \sqrt{n}\}$. More specifically, the ring is enclosed by the circumferences of two circles, each of which has radius $i_x + i - 1$ and $i_x + i - 2$, respectively, at its same center (see Fig. 4). We can see that there exist $\Theta(i_x + i)$ nodes in the layer l'_i since the area of l'_i is given by $\pi(2i_x + 2i - 3)$. Then from (11), the term $d_L^{(i)}$ is given by

$$d_L^{(i)} = \frac{1}{c_0} \sum_{k_x=1}^{\sqrt{n}/2} \sum_{k_y=1}^{\sqrt{n}} \frac{1}{((i_x + k_x - 1)^2 + (i_y - k_y)^2)^{\alpha/2} a(f)^{\sqrt{(i_x + k_x - 1)^2 + (i_y - k_y)^2}}}.$$

It is further assumed that all the nodes in each layer are moved onto the innermost boundary of the corresponding ring, which provides an upper bound for $d_L^{(i)}$. Since there is no node, located on the right half of the cut L , in the first layer l'_1 , $d_L^{(i)}$ is then upper-bounded by

$$\begin{aligned} d_L^{(i)} &\leq \frac{1}{c_0} \sum_{k'=i_x}^{\infty} \frac{c_9 k'}{k'^{\alpha} a(f)^{k'}} \\ &\leq \frac{c_9}{c_0 i_x^{\alpha-1}} \sum_{k'=i_x}^{\infty} \frac{1}{a(f)^{k'}} \\ &\leq \frac{c_9}{c_0 i_x^{\alpha-1}} \left(\frac{1}{a(f)^{i_x}} + \int_{i_x}^{\infty} \frac{1}{a(f)^x} dx \right) \\ &\leq \frac{c_{10}}{i_x^{\alpha-1} a(f)^{i_x}}, \end{aligned}$$

where c_0 , c_9 , and c_{10} are some positive constants independent of n . Here, the fourth inequality holds since $a(f) > 1$. To get a lower bound for $d_L^{(i)}$, nodes in each layer are now moved onto the outermost boundary of the corresponding ring. Let δ_i denote the fraction of nodes that are placed on the right half of the network among nodes in the i -th layer l'_i , which is obviously independent of n irrespective of $i \in \{1, \dots, \sqrt{n}/2\}$. Thus, the lower bound similarly follows

$$\begin{aligned} d_L^{(i)} &\geq \frac{1}{c_0} \sum_{k'=i_x}^{i_x + \sqrt{n}/2 - 1} \frac{c_9 \delta_{k' - i_x + 1} k'}{k'^{\alpha} a(f)^{k'}} \\ &\geq \frac{c_9 \min\{\delta_1, \dots, \delta_{\sqrt{n}/2}\}}{c_0} \sum_{k'=i_x}^{i_x + \sqrt{n}/2 - 1} \frac{1}{k'^{\alpha-1} a(f)^{k'}} \\ &\geq \frac{c_{11}}{i_x^{\alpha-1} a(f)^{i_x}}, \end{aligned}$$

where c_0 , c_9 , and c_{11} are some positive constants independent of n , which finally yields (12). This completes the proof.

B. Proof of Lemma 2

Since the size of matrix \mathbf{F}_L is given by $\Theta(n) \times \Theta(n)$, the analysis essentially follows the argument in [10] with a slight modification (refer to Appendix III in [10] for more precise description). Suppose that $a(f) = \Omega((1 + \epsilon_0)^{\sqrt{n}})$ for an arbitrarily small $\epsilon_0 > 0$. Then in the following, from the result of Lemma 1, we derive $\sum_{k \in D_L} |[\mathbf{F}_L]_{ki}|^2$ and an upper bound for $\sum_{i \in S_L} |[\mathbf{F}_L]_{ki}|^2$:

$$\begin{aligned} \sum_{k \in D_L} |[\mathbf{F}_L]_{ki}|^2 &= \sum_{k \in D_L} \left| \frac{1}{\sqrt{d_L^{(i)}}} [\mathbf{H}_L]_{ki} \right|^2 \\ &= \frac{\sum_{k \in D_L} |[\mathbf{H}_L]_{ki}|^2}{\sum_{k \in D_L} A(r_{ki}, f)^{-1}} \\ &= 1, \end{aligned}$$

where the second equality comes from (1), (8), and (11), and

$$\begin{aligned} \sum_{i \in S_L} |[\mathbf{F}_L]_{ki}|^2 &= \sum_{i \in S_L} \left| \frac{1}{\sqrt{d_L^{(i)}}} [\mathbf{H}_L]_{ki} \right|^2 \\ &\leq c_0 \sum_{i_x=1}^{\sqrt{n}/2} \sum_{i_y=1}^{\sqrt{n}} \frac{i_x^{\alpha-1} a(f)^{i_x}}{((i_x + k_x - 1)^2 + (i_y - k_y)^2)^{\alpha/2} a(f)^{\sqrt{(i_x + k_x - 1)^2 + (i_y - k_y)^2}}} \\ &\leq c_0 \sum_{i_x=1}^{\sqrt{n}/2} \sum_{i_y=1}^{\sqrt{n}} \frac{a(f)^{i_x}}{\sqrt{(i_x + k_x - 1)^2 + (i_y - k_y)^2} a(f)^{\sqrt{(i_x + k_x - 1)^2 + (i_y - k_y)^2}}} \\ &\leq c_0 \sum_{i_x=1}^{\sqrt{n}/2} \frac{1}{i_x + k_x - 1} \left(\sum_{i_y=1}^{\sqrt{n}} \frac{1}{a(f)^{\sqrt{i_x^2 + i_y^2 - i_x}}} \right) \\ &\leq c_0 \sum_{i_x=1}^{\sqrt{n}/2} \frac{1}{i_x} \left(\sum_{i_y=1}^{\sqrt{n}} \frac{1}{a(f)^{i_y^2 / (\sqrt{i_x^2 + i_y^2 - i_x})}} \right) \\ &\leq c_0 \sum_{i_x=1}^{\sqrt{n}/2} \frac{1}{i_x} \left(\sum_{i_y=1}^{\sqrt{n}} \frac{1}{(1 + \epsilon_0)^{c_{12} i_y^2}} \right) \\ &\leq c_0 \left(1 + \int_1^{\sqrt{n}/2} \frac{1}{x} dx \right) \left(\frac{1}{(1 + \epsilon_0)^{c_{12}}} + \int_1^{\sqrt{n}} \frac{1}{(1 + \epsilon_0)^{c_{12} y}} dy \right) \\ &\leq c_{13} \log n \end{aligned}$$

for some positive constants c_{12} and c_{13} independent of n , where the second inequality holds since $\alpha = 1$ provides the highest upper bound for all $1 \leq \alpha \leq 2$. The fifth inequality comes from the fact that $a(f) = \Omega((1 + \epsilon_0)^{\sqrt{n}})$. Hence, it is proved that both scaling results are the same as the regular network case shown in [10].

Now we are ready to prove the inequality in (14). Following the same line as that in Appendix III of [10], we thus have

$$E \left[\text{tr} \left((\mathbf{F}_L^H \mathbf{F}_L)^q \right) \right] \leq C_q n (c_{14} \log n)^q,$$

where $C_q = \frac{(2q)!}{q!(q+1)!}$ is the Catalan number for any q and $c_{14} > 0$ is some constant independent of n . Then, from the property $\|\mathbf{F}_L\|_2^2 = \lim_{q \rightarrow \infty} \text{tr}((\mathbf{F}_L^H \mathbf{F}_L)^q)^{1/q}$ (refer to [36]), the expectation of the term $\|\mathbf{F}_L\|_2^2$ is

upper-bounded by

$$\begin{aligned} E [\|\mathbf{F}_L\|_2^2] &\leq \lim_{q \rightarrow \infty} (E [\text{tr} ((\mathbf{F}_L^H \mathbf{F}_L)^q)])^{1/q} \\ &\leq \lim_{q \rightarrow \infty} (C_q n (c_{14} \log n)^q)^{1/q} \\ &= 4c_{14} \log n, \end{aligned}$$

where the equality holds since $\lim_{q \rightarrow \infty} C_q^{1/q} = 4$. Here, the first inequality comes from dominated convergence theorem and Jensen's inequality. This completes the proof.

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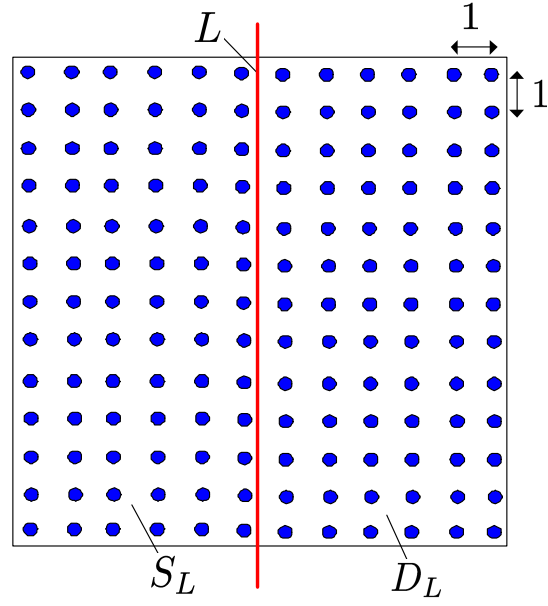


Fig. 1. The cut L in a two-dimensional extended regular network. S_L and D_L represent the sets of source and destination nodes, respectively.

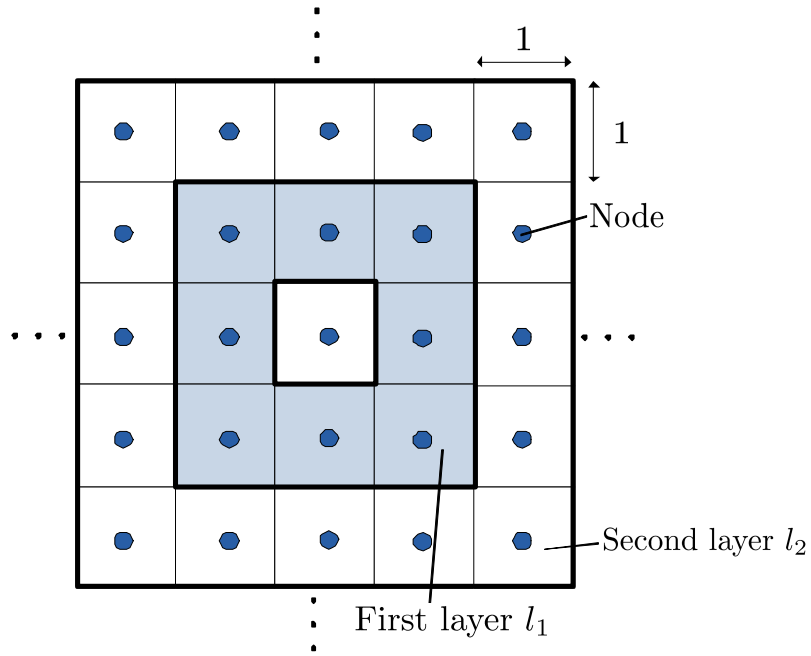


Fig. 2. Grouping of interference routing cells in extended networks. The first layer l_1 represents the outer 8 shaded cells.

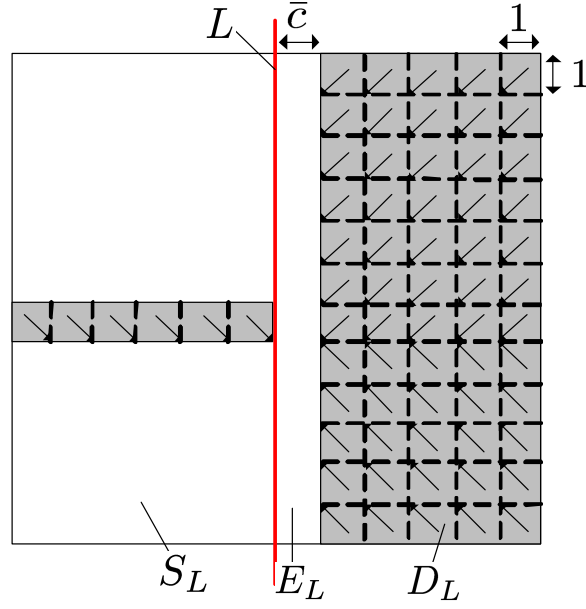


Fig. 3. The node displacement to square vertices, indicated by arrows. The empty zone E_L with width constant \bar{c} is assumed for simplicity.

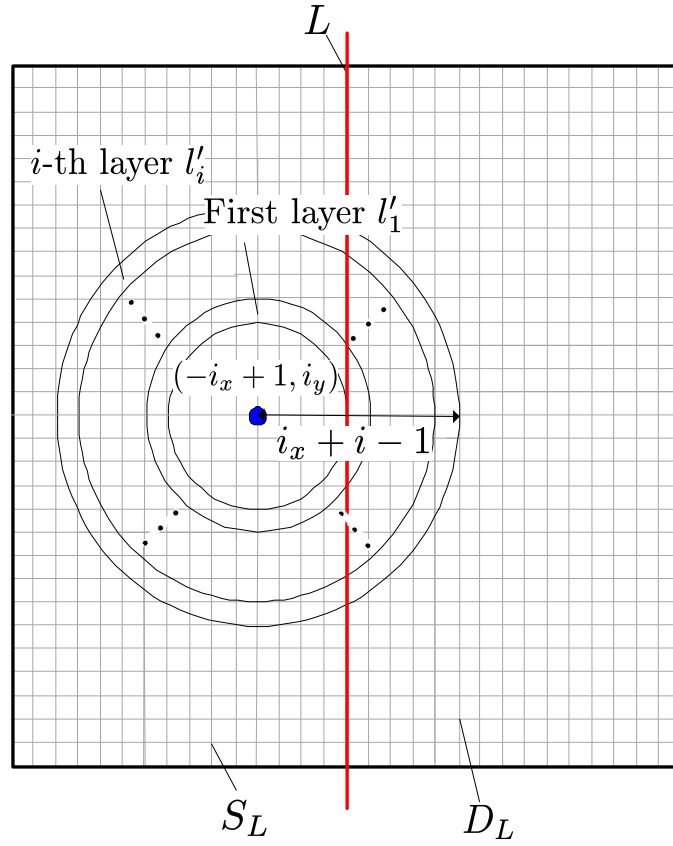


Fig. 4. Grouping of destination nodes in extended networks. There exist $\Theta(i_x)$ nodes in the first layer l'_1 . This figure indicates the case where one source is located at the position $(-i_x + 1, i_y)$. The destination nodes are regularly placed at spacing 1 on the right half of the cut L .